Functional Analysis, PhD in Statistics, A.A. 2022-2023

teacher Annalisa Cesaroni

SCHEDULE.

- wednesday October 19, 14.30-17.30,
- thursday October 20, 10-13,
- wednesday October 26, 14.30-17.30,
- thursday October 27, 10-13,
- wednesday November 2, 14.30-17.30,
- thursday Novermber 3, 10-13,
- wednesday November 9, 14.30-17.30,
- thursday Novermber 10, 10-13.

PRELIMINARY PROGRAM.

Measure theory and integration.

- Definition of σ-algebras, definition of measures, measure spaces. Completion of a σ-algebra.
- Borel σ- algebras and Borel measures. Characterization of σ-finite Borel measures on $\mathbb{R}$ in terms of the cumulative distribution function, the Lebesgue measure on $\mathbb{R}$.
- Measurable functions, in particular Lebesgue measurable functions and random variables. Definition of the Lebesgue integral.
- Singular measures with respect to the Lebesgue measure. Absolutely continuous measure with respect to Lebesgue measure. Density of an absolutely continuous measure. The Lebesgue-Radon-Nikodym decomposition. Distribution of random variables (discrete and continuous).

Hilbert and Banach spaces.

- L^p spaces and spaces of random variables with finite p-moment. Definition of Banach spaces, norms, metric structure induced by the norm. Young inequality, Hölder inequality, Minkowski inequality, with applications, e.g. boundedness of moments of a random variable.
- Bounded linear operators.
- Hilbert spaces, theorem of orthogonal projection and conditional expectation. Orthonormal basis of a Hilbert space, computation of the orthogonal projection. Linear least square estimator.
- Bounded linear operators in a Hilbert space, adjoint of an operator, eigenvalues, spectrum. Spectral theorem for compact symmetric operators, Hilbert-Schimdt operators.

Textbook.

- Lecture notes by the teacher (and references therein).
- G. B. Folland *Real Analysis: modern tecniques and their applications.* Wiley 1999 (2nd ed)