Functional Analysis, PhD in Statistics, A.A. 2021-2022

teacher Annalisa Cesaroni

DETAILED PROGRAM.

Measure theory and integration.

- Definition of σ-algebras, definition of measures, measure spaces. Equivalente definition of measure. Completion of a σ-algebra.
- Borel σ- algebras and Borel measures. Characterization of all Borel measures on $\mathbb{R}$. The Lebesgue measure on $\mathbb{R}$. Example of a set with positive Lebesgue measure which does not contain any interval.
- Measurable functions, in particular Lebesgue measurable functions and random variables.
- Definition of the Lebesgue integral.
- Singular measures with respect to the Lebesgue measure. Example: Dirac measure and counting measure. Absolutely continuous measure with respect to Lebesgue measure. Characterization of σ-finite absolutely continuous measures with respect to Lebesgue in terms of positive L^1_{loc} functions. The Lebesgue-Radon-Nikodym decomposition.
- Distribution of random variables (discrete and continuous).

Hilbert and Banach spaces.

- Banach spaces and metric structure induced by the norm.
- Bounded linear operators.
- Spaces of integrable functions, L^p spaces. The Young inequality and the Hölder inequality, and the Minkowski inequality with applications, e.g. boundedness of moments of a random variable.
- Hilbert spaces, theorem of orthogonal projection and orthonormal basis. Hilbert-Schimdt operators.
- Some example of Hilbert spaces: L^2 and H^1 (with definition of weak derivative). Some example of Hilbert-Schimdt operators.

Textbook.

- Lecture notes by the teacher (and references therein).
- G. B. Folland *Real Analysis: modern tecniques and their applications.* Wiley 1999 (2nd ed)