Course unit English denomination	Functional Analysis
Teacher in charge (if defined)	Annalisa Cesaroni
Teaching Hours	22
Number of ECTS credits allocated	3
Course period	11/2024-01/2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ Yes (100% minimum of presence, apart from exceptional absences that must be justify in advance) ☐ No
Course unit contents	**Measure theory and integration.** Definition of σ-algebras, definition of measures, and measure spaces. Borel σ- algebras and Borel measures. Characterization of σ-finite Borel measures on R in terms of the cumulative distribution function. Definition of the Lebesgue measure on R and R^n. Measurable functions, and random variables. Definition of the Lebesgue integral. Singular measures and absolutely continuous measures with respect to the Lebesgue measure. Density of an absolutely continuous measure. The Lebesgue-Radon-Nikodym decomposition theorem, differentiation of measures. Distribution of random variables (discrete and continuous). **Banach spaces.** L^p spaces and spaces of random variables with finite p-moment. Definition of Banach spaces, norms, metric structure induced by the norm. Young inequality, Holder inequality, Minkowski inequality, with applications, e.g. boundedness of moments of a random variable. Bounded linear operators. **Hilbert spaces.** Hilbert spaces, theorem of orthogonal projection and conditional expectation. Orthonormal basis of a Hilbert space, computation of the orthogonal projection. Linear least square estimator. Fourier series and Fourier transform in L^2. Bounded linear operators in a Hilbert space, adjoint of an operator, eigenvalues, spectrum. Spectral theorem for compact symmetric operators, Hilbert-Schimdt operators. Notion of weak derivative. The Sobolev space H^1.
Learning goals	The aim of the course is to provide basic notions and tools in the (infinite-dimensional) setting of Linear Functional Analysis. The students will acquire knowledge and understanding of many basic tools which are of common use in the analysis of infinite dimensional vector spaces (e.g. the theory of Banach and Hilbert spaces, of linear, bounded, and compact operators), which are of fundamental importance in many branches of applied mathematics, in particular in probability theory and statistics. Moreover the students will be able to solve simple problems requiring manipulation or application of the concepts and results introduced in this course and apply their knowledge in mathematical and statistical domains where functional analytic techniques are relevant.
Teaching methods	• Lectures

Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No Students from other PhD courses may be admitted subject to CV evaluation by the Faculty Board
Prerequisites (not mandatory)	Basics of linear algebra. Basics of calculus.
Examination methods (in applicable)	Written exam on the contents of the course
Suggested readings	• Course material available from the instructor • A. Bressan Lecture notes in Functional Analysis with application to linear partial differential equations . AMS 2012. • G. B. Folland Real Analysis: modern tecniques and their applications. Wiley 1999 (2nd ed)
Additional information	