Prerequisites for Admission to the PhD Course in Statistical Sciences University of Padova

Candidates for the PhD course in Statistical Sciences are required to be sufficiently well trained to be able to benefit from the study and research activities planned for the programme. At the start of the programme, it will be assumed that students have a level of preparation equivalent to at least 5 university courses of: mathematics (including linear algebra), probability and statistics. Furthermore, a working knowledge of English will be required.
The necessary prerequisites required at the start of the programme are fundamental mathematical skills and basic knowledge in probability and statistics as described below.

Mathematics

Linear Algebra: Vector spaces, linear dependence, bases and coordinate systems, scalar product and orthogonality. Linear transformations and matrices, solution to linear systems, projection matrices, eigenvectors and eigenvalues, symmetric and positive definite matrices, diagonalization, determinants.

Calculus: Trigonometric and trascendental functions. Sequences of real numbers and series, notion of distance in $\mathbf{R}$ and $\mathbf{R}^2$, limits and continuity. Differential calculus: notion of derivative, mean value theorem and Taylor polynomials. Differential calculus for functions in several variables: gradient, differential, Hessian matrix. Extreme values and classification of critical points. Constrained extreme values: linear and nonlinear programming and the Lagrange multipliers method. Integral calculus and the fundamental theorem of calculus. Double integrals.

Bibliographic references:

Robert A. Adams and Christopher Essex, *Calculus: A Complete Course,* 9th ed., Pearson Press, 2017.

Tom M. Apostol, *Calculus, vol. 1, vol. 2*, Wiley Press, 1961, 1969.

Gilbert Strang, *Introduction to Linear Algebra*, 5th ed., Wellesley - Cambridge Press, 2016. Web page: `http://math.mit.edu/~gs/linearalgebra/`.

Basic Probability

Probability spaces, conditional probability and independence, Bayes's formula. Random variables, expected value, variance, cumulative distribution, characteristic function and moment generating function associated to a random variable. Discrete random variables: Bernoulli, binomial, Poisson, geometric, hypergeometric distributions. Continuous random variables: uniform, gamma, exponential, normal distributions. Jointly distributed random variables, independent random variables, covariance and correlation, marginal and

conditional distributions. Random vectors and multivariate normal distributions. Convergence of sequences of random variables (weak, in probability, strong). Limit theorems: central limit theorem, weak and strong law of large numbers.

Bibliographic reference:

Sheldon M. Ross, *A First Course in Probability*, 9th ed., Pearson Prentice Hall, 2014.

Basic Statistics

Frequency distributions. Measures of location, variability and shape. Correlation and regression. Population and sample. Statistical models and main examples of parametric models (binomial, Poisson, multinomial, normal, beta, gamma, multivariate normal). Sufficient statistics and Neyman–Fisher factorization theorem. Point estimation: methods of estimation, sampling distribution of an estimator. Confidence intervals and regions. Hypothesis testing: test statistic, p-value, power function, Neyman–Pearson lemma, relationship with confidence intervals. Goodness of fit tests and contingency tables. The likelihood function, score, observed and expected information and main properties. Estimation, tests and confidence intervals based on the likelihood function. Standard inference problems about parameters of normal models (including t and F tests, and multiple linear regression with matrix notation), binomial models, Poisson models, multinomial models.

Bibliographic references:

George Casella and Roger L. Berger, *Statistical Inference*, 2nd ed., Duxbury, 2001.

Morris H. De Groot and Mark J. Schervish, *Probability and Statistics*, 4th ed., Addison–Wesley, 2012.

John A. Rice, *Mathematical Statistics and Data Analysis*, 3rd ed., Duxbury, 2007.